PHS 106

PHYSICS FOR AGRICULTURE AND BIOLOGICAL STUDENTS

II

J. O. AKINLAMI Ph.D.
Course Outline

PART ONE: WAVES AND OPTICS

1. Waves
2. Optics
CHAPTER ONE

1.0 WAVES

This is the transmission of vibration (energy) through a material medium or vacuum. A wave allows energy to be transferred from one point to another some distance away without any particles of the medium traveling between the two points.

1.1 Wave Motion

The motion through a medium of any quantity (such as displacement, velocity, pressure, electric or magnetic fields) that varies with time. For example, the circular ripples produced on the surface of a pool when a stone is dropped on the surface of a water. Sound wave can propagate only through a material (elastic) medium e.g. air, water, wood or metal, there is no sound from an electric bell ringing in vacuum or free space. In contrast electromagnetic (E-M) waves e.g. heat or light waves or x rays can propagate in material medium as well as in vacuum we note that the production of sound requires a vibrating source, a medium and a receiver. There exist transverse and longitudinal waves and waves which are a mixture of these.

1.2 Types of Wave

1.2.1 Transverse Waves

This is a wave which is propagated by vibrations perpendicular to the direction of travel of the wave. Examples include waves on plucked strings and on water, electromagnetic waves which include light waves.

1.2.2 Longitudinal Waves

This is a wave in which the vibrations occur in the same direction as the direction of travel of the wave. The most common example of a longitudinal wave is a sound wave.

1.3 Progressive Wave

A wave that spread out continuously from a vibrator is called a progressive wave. For example, both transverse and longitudinal waves are progressive. This means that the wave profile moves along with the speed of the wave. A progressive repeats at equal distance. The repeat distance is the wavelength λ. If one point is taken and the profile is observed as it passes this point, then the profile is seen to repeat at equal intervals of time. The repeat time is the period T.

The vibrations of the particles in a progressive wave are of the same amplitude and frequency. But the phase of the vibrations changes for different points along the wave.

1.3.1 Progressive Wave Equation

Let us suppose that the wave moves from left to right and that a particle at the origin then vibrates according to the equation \( y = a \sin \omega t \), where t is the time and \( \omega = 2 \pi f \), y is the varying quantity in the displacement (y) of the particle. In general y will depend on the position \( n \) the medium and time, \( y = y(x,t) \).

If we consider a particle say P at a distance x from origin to the right, the phase of the vibration will be different from that at origin. A distance \( \lambda \) from origin corresponds to a phase difference of \( 2\pi \). Thus the phase difference \( \varphi \) at x is given by
\[ \frac{2\pi x}{\lambda} \]. Hence, the displacement of any particle at a distance \( x \) from the origin is given by

\[
y = A\sin(\omega t - \phi)
\]

\[
y = A\sin\left(\omega t - \frac{2\pi x}{\lambda}\right)
\]

\( \omega = 2\pi f \), \( \omega = \frac{2\pi v}{\lambda} \), \( v \) is the velocity of the wave

\[
y = A\sin\left(\frac{2\pi vt}{\lambda} - \frac{2\pi x}{\lambda}\right)
\]

\[
y = A\sin\left(\frac{2\pi}{\omega}(vt - x)\right), \text{ we know that } \omega = \frac{2\pi}{T},
\]

\[
y = A\sin 2\pi\left(\frac{t}{T} - \frac{x}{\lambda}\right)
\]

We have negative sign because the wave moves from left to right but if the wave moves from right to the left which is the opposite direction, so the wave equation is given by

\[
y = A\sin 2\pi\left(\frac{t}{T} + \frac{x}{\lambda}\right)
\]

In general, \( y = A\sin 2\pi\left(\frac{t}{T} \pm \frac{x}{\lambda}\right) \)

### 1.4 Properties of Wave

#### 1.4.1 Reflection

Any wave motion can be reflected. Light waves and sound waves are reflected from a plane surface so that the angle of incidence is equal to the angle of reflection.

#### 1.4.2 Refraction

This is the change in direction of a wave when it enter a new medium. This is due to the change in velocity of the wave on entering a different medium. We know that light wave is refracted likewise sound waves are also refracted.

The refraction of sound explains why sounds are easier to hear at night than during day time. In the day time, the upper layers of air are colder than the layers near the earth. Now sound travels faster the higher the temperature and sound waves are hence refracted in a direction away from the earth. The intensity of the sound waves
thus diminishes. At night time, however, the layers of air near the earth are colder than those higher up and hence sound waves are now refracted towards the earth with a consequent increase in intensity.

1.4.3 Diffraction
This is the spreading out of waves when they pass through apertures or around obstacles. This property of wave depend on the width of the aperture in relation to the wavelength, so, the smaller the width of the aperture in relation to the wavelength, the greater is the spreading or diffraction of the waves. Light waves, electromagnetic waves and sound waves are all diffracted. For diffraction to occur the opening of the aperture must be of the order of the magnitude of the wavelength of the wave.

1.4.4 Interference
This is the property of wave in which two or more waves of the same frequency overlap.

1.4.5 Polarization

1.5 Velocity of Wave

1.5.1 Transverse Wave on String

\[ v = \sqrt{\frac{T}{m}} \]

T is the tension in the string and m is mass per unit length of the string.

1.5.2 Sound Waves in Gas

\[ v = \sqrt{\frac{\gamma P}{\rho}} \]

P is the pressure, ρ is the density and γ is the ratio of the molar heat capacity of the gas.

For a gas of mass m and volume V, \( \rho = \frac{m}{V} \), hence,

\[ v = \sqrt{\frac{\gamma PV}{m}} \]

But for a gas, \( PV = \frac{mRT}{M} \), where R is the universal gas constant pr unit mass of the gas and T is absolute temperature. Thus, \( \frac{PV}{m} = \frac{RT}{M} \). Hence,
\[ v = \sqrt{\frac{\gamma RT}{M}} \]

since \( \gamma \) and \( R \) are constant for a given gas we see that velocity is independent of pressure \( P \) if temperature \( T \) remain constant. This is confirmed by experiment (the speed of sound at the top of mountain is approximately equal to the speed at the bottom) in particular note that

\[ v \propto \sqrt{T} \]

1.5.3 Longitudinal Waves in Solid

\[ v = \sqrt{\frac{E}{\rho}} \]

\( E \) is young modulus and \( \rho \) is the density of solid.

1.5.4 Electromagnetic Waves

\[ v = \sqrt{\frac{1}{\mu \epsilon}} \]

\( \mu \) is the permeability and \( \epsilon \) is the permittivity of the medium.

Example 1

Calculate the speed of sound in steel of young modulus \( 2 \times 10^{11} \text{Nm}^{-2} \) and with density \( 7800 \text{kgm}^{-3} \).

Solution

\[ v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{2 \times 10^{11}}{7800}} = 5060 \text{ms}^{-1} \]

Example 2

Compute the speed of longitudinal wave air at \( 27^\circ \text{C} \). If its mean molecular mass is \( 28.8 \times 10^{-3} \text{kgmol}^{-1} \) and given that \( \gamma \) is 1.40 and \( R = 8.314 \text{Jmol}^{-1}\text{K}^{-1} \).

Solution

\[ v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{1.40 \times 8.314 \times 300}{28.8 \times 10^{-3}}} = 348 \text{ms}^{-1} \]
1.6 Stationary Waves
A stationary wave is always set up when two plane progressive wave of equal amplitude and frequency travel in opposite direction in the same medium.

1.7 Beats
If two notes of nearly equal frequency are sounded together, a periodic rise and fall in intensity can be heard. This is known as the phenomenon of beats. The frequency of the beats is the number of intense sounds heard per second.

1.7.1 Beat Frequency Formula
Let us suppose that two sounding tuning forks have frequencies \( f_1 \), \( f_2 \) cycles per second which are close to each other. At some instant the displacement of a particular layer of air near the ear due to each fork will be a maximum to the right. The resultant displacement is then a maximum, and a loud sound or beat is heard. The vibrations of air after this, due to each fork go out of phase, and \( t \) seconds later the displacement due to each fork is again a maximum to the right, so that loud sound or beat is heard again. One fork has then made exactly one cycle more than the other. The number of cycles made by each fork in \( t \) seconds is \( f_1t \) and \( f_2t \) respectively. If we assume that \( f_1 \) is greater than \( f_2 \), we therefore have

\[
\frac{f_1t - f_2t}{t} = 1
\]

\[
f_1 - f_2 = \frac{1}{t}
\]

Now 1 beat has been made in \( t \) seconds, so that \( \frac{1}{t} \) is the number of beats per second or beat frequency. Therefore,

\[
f_1 - f_2 = \text{beat frequency}
\]

\[
f = f_1 - f_2
\]

1.7.2 Uses of Beats
1. It is used to measure the unknown frequency \( f_1 \) of a note. A note of known frequency \( f_2 \) is used to provide beats with the unknown note, and the frequency \( f \) of the beats is obtained by counting the number made in a given time. Since \( f \) is the difference between \( f_2 \) and \( f_1 \), it follows that \( f_1 = f_2 - f \) or \( f_1 = f_2 + f \).
Let \( f_2 = 1000\text{Hz} \) and the number of beats per second made with a tuning fork of unknown frequency \( f_1 \) is 4. Then \( f_1 = 1004 \) or \( 996\text{Hz} \).

2. Beats are also used to tune an instrument to a given note. As the instrument note approaches the given note, beats are heard.
1.8 Doppler Effect

This is the change in apparent frequency of a source (of sound or light) due to relative motion of the source and the observer.

If a source of sound is moving relative to an observer a rise or fall of pitch is heard by the observer according to whether the distance between them is decreasing or increasing this is Doppler Effect.

If \( v \) represent velocity of sound in air, \( u_s \) velocity of source of sound \( u_o \) velocity of an observer, \( f \) the true frequency of the source, \( v' \) represent apparent velocity, \( \lambda' \) represent apparent wavelength, therefore the apparent frequency \( f' \) is given by

\[
f' = \frac{v}{\lambda}
\]

Case I: Source Moving Toward Stationary Observer

Apparent velocity \( v' = v - u_s \) because the source moved towards the observer. Thus the apparent wavelength \( \lambda' \) of the waves reaching observer is

\[
\lambda' = \frac{v - u_s}{f}
\]

Therefore, apparent frequency \( f' = \frac{v}{\lambda} \)

\[
f' = \left( \frac{v - u_s}{v - u_s} \right) f
\]

\[
f' = \left( \frac{v}{v - u_s} \right) f
\]

\( v - u_s \) is less than \( v - u_s \), \( f' \) is greater than \( f \), the apparent frequency thus appears to increase when a source is moving towards an observer.

Case II: Observer and Source Moves Towards Each Other and Wave moves in the same direction as Source

The apparent velocity \( v' = v + u_o \). Thus the apparent wavelength \( \lambda' \) of the waves is

\[
\lambda' = \frac{v - u_s}{f}
\]

The apparent frequency \( f' = \frac{v}{\lambda} \)

\[
f' = \left( \frac{v + u_o}{v - u_s} \right) f
\]
It should be noted that the motion of the observer affects only $v'$, the velocity of the waves reaching the observer, while the motion of the source affects only $\lambda'$, the wavelength of the waves reaching the observer.

**Case III: Source Moving Away from Stationary Observer**

The apparent velocity $v' = v + u_o$. Thus, the apparent wavelength $\lambda'$ of the waves reaching observer is

$$\lambda' = \frac{v + u_s}{f}$$

Hence, the apparent frequency $f'$ is $f' = \frac{v'}{\lambda'}$

$$f' = \left( \frac{v + u_o}{v + u_s} \right) f$$

$v + u_s$ is greater than $v + u_o$, $f'$ is less than $f$, and hence the apparent frequency decreases when a source moves away from an observer.

**Case IV: Source Stationary and Observer moving Towards It**

The apparent velocity $v' = v + u_o$. Thus the apparent wavelength $\lambda'$ of the wave reaching observer is

$$\lambda' = \frac{v}{f}, \text{ since } u_s = 0$$

Hence, the apparent frequency $f'$ is $f' = \frac{v'}{\lambda'}$

$$f' = \left( \frac{v + u_o}{v} \right) f$$

$v + u_o$ is greater than $v$, $f'$ is greater than $f$, thus the apparent frequency is increased.

**Case IV: Source Stationary and Observer moving Away from It**

The apparent velocity $v' = v - u_o$. Thus, the apparent wavelength $\lambda'$ of the wave reaching observer is $\lambda' = \frac{v}{f}$

Hence, the apparent frequency $f'$ is $f' = \frac{v'}{\lambda'}$

$$f' = \left( \frac{v - u_o}{v} \right) f$$

$v - u_o$ is less than $v$, the apparent frequency $f'$ appears to be decreased.
CHAPTER TWO

2.0 OPTICS

2.1 Reflection at Plane Surfaces

It is well known that highly polished metal surfaces reflect about 80 to 90 per cent of the light incident on them. So as a result mirrors are made or produce by depositing silver on the back of glass. At times the front of the glass is coated with the metal, as an example the largest reflector in the world is a curved mirror early 5 metres across, the front of which is coated with Aluminium.

2.1.1 Laws of Reflection

First law states that the reflected ray, the incident ray and the normal to the mirror at the point of incidence all lie in the same plane.

Second law states that the angle of incidence, $i$ is equal to the angle of reflection, $r$. That is, $i = r$.

2.2 Deviation of Light by Plane Mirror

Plane mirror is use in a simple periscope to change or deviate light from one direction to another.

Figure 2.1 Deviation of Light by Plane Mirror

Let us consider a ray PO incident at O on a plane mirror MN. The angle POM made by PO with MN is known as the glancing angle, $g$ with the mirror, and since the angle of reflection is equal to the angle of incidence, the glancing angle QON made by the reflected ray OQ with the mirror is also equal to $g$.

The light has been deviated from a direction PO to a direction OQ. Since angle RON = angle MOP = $g$, it follows that, angle of deviation $d = 2g$. So, in general, the angle of deviation of a ray by a plane surface is twice the glancing angle.

2.3 Reflection at Curved Surfaces

Curved surfaces or mirror are cut out from spherical surfaces with a radius of curvature. Curved mirrors are used in motor cars as driving mirrors. We have two types of curved mirror, convex and concave mirrors. Convex mirror is referred to as diverging mirror while concave mirror is referred to as converging mirror.
2.3.1 Focus of a Concave Mirror
Beam of light incident on a concave mirror and parallel to the principal axis and close to it are all reflected and they converge to a point F on the principal axis, which is known as the principal focus of the mirror.

![Figure 2.2 Focus of a Concave Mirror](image)

Figure 2.2 Focus of a Concave Mirror

P is the midpoint of the mirror and it is called its pole. C is the centre of the sphere of which the mirror is part and it is known as the centre of curvature. Line PC is known as the principal axis and rays of light parallel to PC are called secondary axes.

2.3.2 Focus of a Convex Mirror
Beam of light incident on a convex mirror and parallel to the principal axis and close to it are all reflected and the reflected rays form a divergent beam which appears to come from a point F behind the mirror. Hence, a convex mirror has a virtual focus.

![Figure 2.3 Focus of a Convex Mirror](image)

Figure 2.3 Focus of a Convex Mirror
Radius of Curvature (r): This is the distance PC from the pole to the centre of curvature.

Focal length (f): This is the distance PF from the pole to the focus.

Radius of curvature r, and focal length f, are related by the formula \( r = 2f \)

\[ f = \frac{r}{2} \]

This holds for concave and convex mirror.

### 2.3.3 Curved Mirror Formula

\[ \frac{1}{v} + \frac{1}{u} = \frac{1}{f} \]

where \( u \) is object distance, \( v \) is the image distance and \( f \) is the focal length.

Sign convention is very important in the use of optical (mirror) formula. A real object or image distance is a positive distance while a virtual object or image distance is a negative distance. This means ‘real’ is positive and ‘virtual’ is negative. Therefore, the focal length of a concave mirror is a positive distance while the focal length of a convex mirror is a negative distance.

### 2.3.4 Magnification

The lateral magnification, \( m \), produced by a mirror is defined by

\[ m = \frac{\text{Image height}}{\text{Object height}} \]

\[ m = \frac{IH}{OH} \]

This is also equivalent to

\[ m = \frac{IH}{OH} = \frac{v}{u} \]
Example

An object is placed 15cm in front of a concave mirror of focal length 20cm. Find the image position and the magnification.

Solution

\[
\frac{1}{v} + \frac{1}{u} = \frac{1}{f}
\]

\[
\frac{1}{v} + \frac{1}{15} = \frac{1}{20}
\]

\[
\frac{1}{v} = \frac{1}{20} - \frac{1}{15}
\]

\[
\frac{1}{v} = \frac{3 - 4}{60}
\]

\[
\frac{1}{v} = \frac{-1}{60}
\]

\[
v = -60cm.
\]

Since \(v\) is negative, it means the image is virtual and it is 60cm from the mirror.

Magnification \(m = \frac{v}{u} = \frac{60}{15} = 4\)

This means the image is four times as high as the object.

2.4 Refraction at Plane Surface

2.4.1 Laws of Refraction

First law states that the incident and refracted rays and the normal at the point of incidence, all lie in the same plane.

Second law states that for any two given media, the ratio of sine of angle of incidence to sine of angle of refraction \(\left(\frac{\sin i}{\sin r}\right)\) is a constant. Where \(i\) is the angle of incidence and \(r\) is the angle of refraction.
2.4.2 Refractive Index

This is the constant ratio \( \frac{\sin \theta_1}{\sin \theta_2} \).

Light is refracted because it has different velocities in different medium. The wave theory of light shows that the refractive index for two given media 1 and 2 is given by

\[
\eta_1 \eta_2 = \frac{\text{velocity of light in medium 1}}{\text{velocity of light in medium 2}}
\]

For absolute refractive index, \( \eta \), of a medium we have

\[
\eta = \frac{\text{velocity of light in a vacuum, } c}{\text{velocity of light in medium, } v}
\]

2.4.3 Relation between Refractive Indices

![Diagram](image)

Figure 2.4 Relation between Refractive Indices

Let us consider a ray of light NO, refracted from glass to air along the direction OM in the figure above. It was observed that the refracted ray OM bent away from the normal. The refractive index from glass to air is given by

\[
\eta_a = \frac{\sin \theta_1}{\sin \theta_2}
\]

where \( \theta_1 \) is the angle of incidence in the glass and \( \theta_2 \) is the angle of refraction in the air.
From the principle of reversibility of light, it follows that a ray traveling along MO in air is refracted along ON in the glass. The refractive index from air to glass is then given by

$$\eta_g = \frac{\sin q}{\sin p}$$

But

$$\eta_a = \frac{\sin p}{\sin q}$$

$$\therefore \eta_a = \frac{1}{\eta_g}$$

Therefore, if $$\eta_g = 1.5$$, then $$\eta_a = \frac{1}{1.5} = 0.67$$

### 2.5 Total Internal Reflection

![Figure 2.5A](image_url)
Figure 2.5 Total Internal Reflection
Let us consider a ray KO in glass which is incident at a small angle i on a glass – air plane boundary, it was observed that part of the incident light is reflected along OM in the glass, while the remainder of the light is refracted away from the normal at an angle r into the air. The reflected ray OM is weak, but the refracted ray ON is bright. This means that most of the incident light energy is transmitted and a little is reflected.

On increasing the angle of incidence, i, in the glass, the angle of emergence, r, is also increased and at some angle of incidence C in the glass the refracted ray ON travels along the glass – air boundary, making an angle of refraction of 90°. The reflected ray OM is still weak in intensity, but as the angle of incidence in the glass is increased slightly the reflected ray suddenly becomes bright, and no refracted ray is then observed. Since all the incident light energy is now reflected, total reflection is said to take place in the glass at O.

A critical stage is reached at the point of incidence O when the angle of refraction in air is 90° and the angle of incidence in the glass is known as the critical angle for glass and air.

From figure 2.5B, $\eta = \frac{\sin 90}{\sin C}$

$\eta \sin C = \sin 90$

$\eta \sin C = 1$

$\sin C = \frac{1}{\eta}$

Example

What is the critical angle for a crown glass with a refractive index of about 1.51 for yellow light?

Solution

$\sin C = \frac{1}{\eta}$

$\sin C = \frac{1}{1.51} = 0.667$

$C = \sin^{-1}0.667$

$C = 41.5°$
2.6 Refraction Through Thin Lenses

2.6.1 Converging and Diverging Lenses
A lens is an object, which is made of glass and it is bounded by one or two spherical surfaces. We have two types of lenses; converging and diverging lenses. Converging lens is thicker at the middle than at the edges and diverging lens is thinner at the middle than at the edge.

Principal Axis of a lens is the line joining the centres of curvature of the two surfaces and passes through the middle of the lens.

2.6.2 Focus of Converging Lens
A converging lens brings an incident parallel beam of rays to a principal focus, F, on the other side of the lens when the beam is narrow and incident close to the principal axis.

2.6.3 Focus of Diverging Lens
When a narrow parallel beam, close to the principal axis, is incident on a thin diverging lens, a beam is obtained which appears to diverge from a point F on the same side as the incident beam. F is known as the principal focus of the diverging lens.

2.6.4 Lens Equation
\[
\frac{1}{v} + \frac{1}{u} = \frac{1}{f}
\]
This is the general lens equation. Where u is object distance, v is the image distance and f is the focal length.

Example
An object is placed 12cm in front of a diverging lens of focal length 24cm. Find the image position.

Solution
\[
\frac{1}{v} + \frac{1}{12} = \frac{1}{-24}
\]
\[
\frac{1}{v} = \frac{1}{-24} - \frac{1}{12}
\]
\[
\frac{1}{v} = -1 - \frac{2}{24}
\]
\[
\frac{1}{v} = -\frac{3}{24}
\]
\[
\frac{1}{v} = -\frac{1}{8}
\]

\[v = -8\text{cm}\]

The image is virtual and it is 8cm from the lens.

### 2.6.5 Magnification

\[
m = \frac{\text{Image height}}{\text{Object height}}
\]

\[
m = \frac{IH}{OH}
\]

This is also equivalent to

\[
m = \frac{IH}{OH} = \frac{v}{u}
\]

\[
\frac{1}{v} + \frac{1}{u} = \frac{1}{f}
\]

multiply through by \(v\), we have

\[
1 + \frac{v}{u} = \frac{v}{f}
\]

\[
1 + m = \frac{v}{f}
\]

\[
m = \frac{v}{f} - 1
\]

If we also multiply the lens equation by \(u\), we have
\[
\frac{u}{v} + 1 = \frac{u}{f}
\]

\[
\frac{1}{m} + 1 = \frac{u}{f}
\]

Example

Find the magnification, if a real image is formed 25 cm from a converging lens of focal length 10 cm.

Solution

\[m = \frac{v}{f} - 1\]

\[m = \frac{25}{10} - 1\]

\[m = 2.5 - 1\]

\[m = 1.5\]

2.7 Defects of Vision

Several common defects of vision result from an incorrect relation between the parts of the optical system of the eye. A normal eye form an image on the retina of an object at infinity when the eye is relaxed.

2.7.1 Myopic (Near sighted or Short sighted) Eye

In the myopic eye, the eyeball is too long from front to back in comparison with the radius of curvature of the cornea, and rays from an object at infinity are focused in front of the retina. The most distant object for which an image can be formed on the retina is then nearer than infinity.

2.7.2 Hyperopic (Farsighted or Longsighted) Eye

In the hyperopic eye, the eyeball is too short and the image of an infinitely distant object would be formed behind the retina.

2.7.3 Astigmatism

This refers to a defect in which the surface of the cornea is not spherical, but is more sharply curved in one plane than another. Astigmatism makes it impossible, for example, to focus clearly on the horizontal and vertical bars of a window at the same time.

These defects can be corrected by the use of corrective lenses (glasses). The near point of either a presbyopic or a hyperopic eye is farther from the eye than normal. To see clearly an object at normal reading distance (usually assumed to be
25cm) we must place in front of the eye a lens of such focal length that it forms an image of the object at or beyond the near point. Thus, the function of the lens is not to make the object appear larger, but in effect to move the object farther away from the eye to a point where a sharp retinal image can be formed.

Example

The near point of a certain eye is 100cm in front of the eye. What lens should be used to see clearly an object 25cm in front of the eye.

Solution

\[
\frac{1}{f} = \frac{1}{u} + \frac{1}{v}
\]

\[
\frac{1}{f} = \frac{1}{25} - \frac{1}{100}
\]

\[
\frac{1}{f} = \frac{4 - 1}{100}
\]

\[
\frac{1}{f} = \frac{3}{100}
\]

\[f = +33.3cm\]

That is, a converging lens of focal length 33cm is required. Power of the lens is +3.0 diopter.