EXPERIMENTAL BASIS AND EVIDENCES OF QUANTUM THEORY

Phenomena such as motions of mechanical objects involving distances larger than $10^{-6}$ m can be explained satisfactorily by laws of classical physics which is based on the following basic laws.

i) Newton’s laws of motion

ii) The inverse square law of gravitational attraction between two bodies

iii) Coulomb’s inverse square law of attraction or repulsion between two electrically charged bodies

iv) The law of force on a moving charge in a magnetic field i.e. Lorentz force.

However, certain phenomena could not be explained by classical physics, and due to these failures, Quantum mechanics emerged as a new discipline because of the need to describe these phenomena that could not be explained using Newtonian mechanics or classical electromagnetic theory. These phenomena include the spectral distribution of energy in black body radiation, photoelectric effect, phenomena involving distances of the order of $10^{-10}$ m etc.

The failure of classical physics to explain the distribution of energy in the spectrum of a black body led Max Planck to propose the quantum hypothesis in 1900, which marked the beginning of quantum theory.

**Concept of a Black Body**

A black body is one that absorbs all the radiation (of all wavelengths) that falls on it, and reflects or transmits none. A simple black body can be made by punching a small hole in a box. It is also about the best radiator, as a good absorber its also a good radiator energy. The radiation emitted is called black-body radiation, the full radiation, cavity radiation, or temperature radiation. It has the characteristics feature that the intensity of each frequency has some well-defined value which is determined by the temperature. If the cavity is at (say) 2000K, the small hole emits visible radiation whose colour is characteristic of that temperature. The quality and intensity of the radiation escaping from a black body hole does not depend on the nature of the particular surface from which it escapes but only on its *temperature*. When the body is made hotter, its radiation becomes not only more intense but also more nearly white. When we speak of the quality of
radiation, we mean the relative intensities of the different wavelengths in it, the proportion of red to blue for example.

**Black Body Radiation**

Planck (1900) in an attempt to explain the distribution of energy in the black-body spectrum suggested that when radiation was emitted or absorbed, the emitting or absorbing oscillator always showed a discrete sudden change of energy $\Delta E$. $\Delta E$ is related to the radiation frequency by $\Delta E = h\nu$. $h$ is called the Planck constant $(6.6 \times 10^{-34} \text{Js})$. For visible light, the quantum of energy, or photon carries energy $\Delta E = h\nu = (6.6 \times 10^{-34} \text{Js}) \times (10^{15} \text{s}^{-1}) = 10^{-19} \text{J}$.

The exact value of $\Delta E$ depends on the frequency (colour) of the light.

Einstein (1905) extended Planck’s original idea by suggesting that emissions were not only generated discontinuously, but that they could exhibit particle behavior while being absorbed.

**Properties of blackbody radiation:**

- Radiation emitted by a blackbody is isotropic, homogeneous and unpolarized;
- Blackbody radiation at a given wavelength depends only on the temperature $T$;
- Any two blackbodies at the same temperature emit precisely the same radiation;
- A blackbody emits more radiation than any other type of an object at the same temperature;

**Nature of Radiation:** Thermal Radiation is the energy that travels from one place to another by means of electromagnetic wave motion. When absorbed by matter, it may increase the vibrational or translational kinetic energy of atoms or molecules, this increase of internal energy will usually become apparent as a temperature increases. The vibrational frequencies of atoms at room temperature are about $10^{14} \text{Hz}$. A wave of frequency of about $10^{14} \text{Hz}$ would cause resonance to occur, and in general, would thus be efficient at transferring the electromagnetic wave energy to the electrically charged particles of which matter is composed. Waves whose frequency is close to $10^{14} \text{Hz}$ are called infra-red waves. They are both radiated and absorbed by bodies at normal temperatures. Infra-red radiation is emitted when thermal agitation causes changes in the vibrational and rotational energy states of molecules.
A diathermanous body, such as calcium fluoride prism is one that absorbs little of the radiation passing through it (compare with a body translucent to visible light).

An adiathermanous body, such as a mass of water is one that absorbs strongly the radiation passing through it (compare with a body that is opaque to visible light). For example, a glass is diathermanous when $0.4 \times 10^{-6} \text{m} < \lambda < 2.5 \times 10^{-6} \text{m}$ but diathermanous for longer wavelengths such as $10^{-5} \text{m}$. This fact can be used to explain the action of greenhouse.

**Detection of Radiant Energy:**

Photodetectors depend upon photoelectric effect and photoconductivity. They respond, if a photon of incident radiation has a certain minimum energy (a maximum $\lambda$). Thermal detectors are thermometers, that is, they respond to the temperature changes that accompanies the absorption of all frequencies. The human skin is one example, others include:

i) The sensitive differential air thermometer,

ii) Radiomicrometer, in which a thermocouple is incorporated into a moving-coil galvo suspended by quartz fibre.

iii) The thermopile which consists of about 25 or more thermocouples joined in series, radiation falls on the blackened hot junction, while the cold junction remains shielded.

**DISTRIBUTION OF ENERGY IN THE SPECTRUM OF A BLACK BODY**

To study the quality and distribution of radiant energy from a black body over different wavelengths at constant temperature, Lummer and Pringsheim in 1899 heated a black body
represented by a sphere as shown (in the figure 1, below) to 2000°C and measured its temperature with a thermopile. The beam coming out of the hole was passed through a diffraction grating, which sent the different wavelengths/frequencies in different directions, all towards a screen. To measure the intensities of the various wavelengths, they used an infrared spectrometer and a bolometer.

Each curve gives the relative intensities of the different wavelengths, for a given temperature of the body. The actual intensity of the radiation are shown on the right of the graph, fig 1. The curves showed that as the temperature rises, shorter wavelengths increase more rapidly. Thus the radiation becomes as we have already observed, less red, that is to say, more nearly white. The curve for sunlight has its peak at about 5 x 10⁻⁷m in the visible green, from the position of this peak we conclude that the surface temperature of the sun is about 6000K, see fig 2, stars which
are hotter than the sun, such as Sirius and Vega look blue, not white, because the peaks of their radiation curves lie further towards the visible blue than does the peak of sunlight.

![Graph showing radiation curves of different stars.](image)

Fig 2: shows the surface temperature of the sun to be about 6000K

It is meaningless to speak of the intensity of a single wavelength. The slit of the spectrometer always gathers a band of wavelengths, the narrower the slit, the narrower the band, we therefore always speak of a given band. Hence,

Energy radiated (m$^2$s$^{-1}$) in band $\lambda$ to $(\lambda + \Delta \lambda) = E_\lambda \Delta \lambda$

$E_\lambda$ is called the emissive power of a black body for the wavelength $\lambda$ at the given temperature. Therefore,

$$E_\lambda = \frac{\text{Energy radiated (m}^2\text{s}^{-1}\text{) in band } \lambda \text{ to } (\lambda + \Delta \lambda)}{\text{bandwidth } \Delta \lambda}$$

$$= \frac{\text{Power radiated (m}^2\text{) in band } \lambda \text{ to } (\lambda + \Delta \lambda)}{\text{bandwidth } \Delta \lambda}$$

$E_\lambda$ is expressed in watts per m$^2$ (Wm$^{-2}$) per Angstrom unit (Å) SI unit may be Wm$^{-2}$ per nanometer ($10^{-9}$ m).

The quantity $E_\lambda \Delta \lambda$ is the area beneath the radiation curve between the wavelength $\lambda$ and $\lambda + \Delta \lambda$, (fig 3). Thus the energy radiated per m$^2$ per second between those wavelengths is proportional to that area. Similarly, the total radiation emitted per m$^2$s$^{-1}$ over all wavelengths is proportional to the area under the whole area.
Fig 3: energy radiated per m$^2$ per second between the shown wavelengths

**Laws of Black Body Radiation**

The curves showing the distribution of intensity in Black body radiation can be explained only by Planck’s quantum theory of radiation. Both theory and experiment lead to three generalizations, which together describe well the properties of black body radiation.

1. **Wien's Displacement Law**

   i. Wien's displacement law states that there is an inverse relationship between the wavelength of the peak of the emission of a black body and its temperature when expressed as a function of wavelength, i.e
   \[
   \lambda_m = \frac{b}{T}
   \]

   where $\lambda_m$ is the peak wavelength, $T$ is the absolute temperature of the black body, and $b$ is a constant of proportionality called Wien's displacement constant, equal to $2.8978 \times 10^{-3}$ m·K, that is, at a constant temperature $T$, when wavelength $\lambda$ is increased, the energy emitted $E$, first increases, reaches a maximum and then decreases i.e at a particular temperature, the spectral radiance $E_\lambda$ is a maximum at a particular wavelength $\lambda_m$. 
Fig 4: Wien’s displacement Law

As the temperature increases, the maximum radiancy of energy occurs at shorter wavelength.

ii. If $E_{\lambda m}$ is the height of the peak of the curve for the temperature $T$, then

$$E_{\lambda m} \propto T^5$$

iii. The curve showing the variation of $E_{\lambda}$ with $\Delta \lambda$ at constant temperature $T$ obeys the Planck’s formula,

$$E_{\lambda} = \frac{8\pi hc}{\lambda^5 \left( e^{\frac{hc}{\lambda kT}} - 1 \right)}$$

$$E_{\lambda} = \frac{c_1}{\lambda^5 \left( e^{\frac{hc}{\lambda kT}} - 1 \right)}$$

Where $C_1 = 8\lambda hc$ and $C_2 = \frac{hc}{\lambda}$

2. Stefan’s Boltzmann’s Law

The total radiant energy emitted $E$ per unit time by a black body of surface $A$ is proportional to the fourth power of its absolute temperature.

$$E \propto T^4$$

That is, the total area under each graph of the plots of $E_{\lambda} - \lambda$ graph at temp $T$, fig 3, should be proportional to the corresponding value of $T^4$.

or $E = \sigma AT^4$, $\sigma =$ Stefan's constant $(5.7 \times 10^{-8} \text{Wm}^{-2}\text{K}^{-4})$. 
For a body which is not a black body, then \[ E = \varepsilon \sigma A T^4 \] where \( \varepsilon = \) emissivity of the Black Body, (\( \varepsilon \), emissivity \( = 1 \) for a blackbody)

Note: Emmisitivity and absorptive power have the same value.

**Prevost’s Theory of Heat Exchange**

In 1792, Prevost applied the idea of dynamic equilibrium to radiation. The asserted that a body radiates heat at a rate which depends only on the nature of its surface and its temperature, and that it absorbs heat at a rate depending on the nature of its surface and the temperature of its surroundings.

When the temperature of a body is constant, the body is loosing heat by radiation and gaining it by absorption at equal rates.

In simple terms it means that if a hot body A is placed in an evacuated enclosure B, at a lower temperature. Then A, cools until it reaches the temperature of B. if a body C, cooler then B, is placed in B, then C warms up to the temperature of B. we conclude that radiation from B falls on C, and therefore also on A, even though A is at a higher temperature. Thus A and C each come to equilibrium at the temperature of B when each is absorbing and emitting radiation at equal rates. (Zeroth Law of thermodynamics).

Now, suppose that, after it has reached equilibrium with B, C is transferred from B to a cooler evacuated enclosure D. it loses heat and cools to the temperature of D, therefore, it is radiating heat. However, if C is transferred from B to a warmer enclosure F, then C gains heat and warms up to the temperature of F. It is however unreasonable to suppose that C stops radiating when it is transferred to F. It is more reasonable to suppose that it goes on radiating, but while it is cooler than F, it absorbs more than it radiates.

**Net loss of thermal Energy**

If a body of surface area A is kept at absolute temp T in a surrounding of temperature \( T_0 (T_0 > T) \), then the energy emitted by the body per unit time is:
E = \varepsilon \sigma AT^4

and the energy absorbed per unit time by the body is:

E_0 = \varepsilon \sigma AT_0^4

Net loss of thermal energy per unit time.

\Delta E = E - E_0 = \varepsilon \sigma (T^4 - T_0^4), \text{ but for a Blackbody, } \Delta E = E - E_0 = \sigma A (T^4 - T_0^4).

**Newton's Law of Cooling:**

For a small temperature difference between a body and its surroundings, the rate of cooling of the body is directly proportional to the temperature difference. If a body of temperature \(T\) and surface area \(A\) is kept in a surrounding temperature \(T_0\) (\(T_0 < T\)). Then net loss of thermal energy per unit time.

\[
\frac{dQ}{dt} = \varepsilon \sigma A (T - T_0), \text{ T-temperature of the body, } T_0 - \text{ temperature of the surrounding.}
\]

Since for a cooling body, the rate of heat loss is proportional to the difference in temperature between the body and its surroundings. If the temperature difference is small,

\[T = T_0 + \Delta T\]

\[\Rightarrow \varepsilon \sigma A [(T_0 + \Delta T)^4] = \varepsilon \sigma A [T_0^4 (1 + \frac{\Delta T}{T_0})^4 - T_0^4]\]

\[\Rightarrow \varepsilon \sigma A T_0^4 [1 + 4 \frac{\Delta T}{T_0} + \text{higher powers of } \frac{\Delta T}{T} - 1]\]

\[= 4 \varepsilon \sigma A T_0^3 \Delta T\]

Now, rate of loss of heat at temperature \(T\):

\[\frac{dQ}{dt} = -mc \frac{dT}{dt}\]

\[mc \frac{dT}{dt} = -4 \varepsilon \sigma A T_0^3 [T - T_0]\]
\[
\frac{dT}{dt} = \frac{-4\varepsilon\sigma AT_o^3(T - T_o)}{mC}
\]

\[
\frac{dT}{dt} = -K(T - T_o)
\]

\[K = \frac{4\varepsilon\sigma AT_o^3}{mC}\text{, Note that for a blackbody, } \varepsilon = 1\]

\[
\frac{dT}{dt} \alpha [T - T_o].
\]

3. **Kirchoff’s Law**

Most bodies are coloured, they transmit or reflect some wavelengths better than others. They must absorb these wavelengths weakly, and hence they must also radiate them weakly.

Since most radiation sources are not blackbodies. Some of the energy incident upon them may be reflected or transmitted. The ratio of the radiant emittance \(W'\) of such a source and the radiant emittance \(W\) of a blackbody at the same temperature is called the emissivity \(\varepsilon\) of the source.

That is, \(\varepsilon = W'/W\), see figure 5.

![Diagram showing \(\varepsilon\) of some source](Fig5.png)

This can also be defined thus, the energy falling per \(m^2\) per seconds on the body in the waveband \(\lambda\) to \(\lambda + \Delta\lambda\) is \(E_\lambda\Delta\lambda\), where \(E_\lambda\) is the emissive power of a black body in the neighbourhood of \(\lambda\) at
the temperature of the enclosure. If the body absorbs a fraction $a_\lambda$ of this, we call $a_\lambda$ the spectral absorption factor of the body, for the wavelength $\lambda$. In equilibrium, the body emits as much radiation in the neighbourhood of $\lambda$ as it absorbs; thus

Energy radiated = $a_\lambda E_\lambda \Delta\lambda$ watts per m$^2$

We define the spectral emissivity $e_\lambda$ of a body by the equation

$$e_\lambda = \frac{\text{energy radiated by the body in the range } \lambda \text{ to } \lambda + \Delta\lambda}{\text{energy radiated in the same range by black body at the same temperature}}$$

$$= \frac{\text{energy radiated by the body in the range } \lambda \text{ to } \lambda + \Delta\lambda}{E_\lambda \Delta\lambda}$$

$$= \frac{a_\lambda E_\lambda \Delta\lambda}{E_\lambda \Delta\lambda}$$

$$e_\lambda = a_\lambda = \text{ Kirchhoff's Law}$$

Kirchhoff’s Law: The spectral emissivity of a body for a given wavelength is equal to its spectral absorption factor for the same wavelength.

4. Sun As An Energy Source, Solar Constant:
Solar flux reaching the earth is a function of time determined by
1) the orbital characteristics of the earth and the sun (i.e., eccentricity; obliquity, and periodic precession)
2) the sun properties (e.g., solar surface activity).

NOTE:
a) Sun is a gaseous sphere consisting of hydrogen, helium, iron, silicon, etc.
Solar energy: nuclear fusion (conversion of four hydrogen atoms to one helium atom)
b) Temperature of sun’s photosphere is about 5800 K.
b) Sunspots are cooler regions of the sun (with T = 4000K). Period between sunspot maxima is about 11 years (called 11-year-cycle).

**Solar constant,** $S_0$, is defined as total flux of solar energy, reaching the top of the atmosphere, per unit surface normal to the solar beam at the mean distance between the sun and the earth.

The sun emits about $F_{\text{sun}} = 6.2 \times 10^7$ W/m$^2$. On the basis of energy conservation law, we have

$$F_{\text{sun}} \, 4 \pi r_{\text{sun}}^2 = S_0 \, 4 \pi d_0^2$$

where $r_{\text{sun}}$ is the radius of the sun (6.96x10$^5$ km), and $d_0$ is the mean distance between the sun and the earth (1.5x10$^8$ km). Hence,

$$S_0 = F_{\text{Sun}} (r_{\text{Sun}} / d_0)^2$$

Mean measured value $S_0 = 1366$ W m$^{-2}$ with the measured uncertainty ± 3Wm$^{-2}$

**Actual solar flux at the top of the atmosphere** at a given time is

$$F_\circ = S_0 \left( \frac{d_\circ}{d} \right)^2 \cos(\Theta_\circ)$$

where $d_\circ$ is the mean distance from the center of the sun to the earth and $d$ is the actual distance on a given day (depends of the earth orbit).

**Questions**
1. The tungsten filament of an electric lamp has a length of 0.5 m and a diameter of $6 \times 10^{-5} \text{m}$. The power rating of the lamp is 60 W. Assuming the radiation from the filament is equivalent to 80% that of a perfect black body radiator at the same temperature, estimate the steady temperature of the filament. (Stefan’s constant = $5.7 \times 10^{-8} \text{Wm}^{-2}\text{K}^{-4}$).

**Solution:**

When the temperature is steady,

Power radiated from filament = power received = 60 W

$$0.8 \times 5.7 \times 10^{-8} \times 2\pi \times 3 \times 10^{-5} \times T^4 = 60$$

(Since 80% = 0.8, and surface area of a cylindrical wire is $2\pi r h$)

$$\therefore \quad T = \left(\frac{60}{0.4 \times 5.7 \times 10^{-8} \times 2\pi \times 3 \times 10^{-5}}\right)^{\frac{1}{4}}$$

$$T = 1933 \text{ K}$$

2. A metal sphere with a black surface and radius 30 mm is cooled to $-73^\circ \text{C}$ (200 K) and placed inside an enclosure at a temperature of $27^\circ \text{C}$ (300 K). Calculate the initial rate of temperature rise of the sphere, assuming the sphere is a black body (Assume density of metal = 8000 kg m$^{-3}$, specific heat capacity of metal = 400 J kg$^{-1}$ K$^{-1}$, and Stefan’s constant = $5.7 \times 10^{-8} \text{Wm}^{-2}\text{K}^{-4}$).

**Solution:**

Energy per second radiated by sphere = $\sigma A (T^4 - T_0^4)$

Where $A$ – the surface area ($4\pi r^2$) of the sphere of radius r,

$T = 200 \text{ K}$, and $T_0 = 300 \text{ K}$

Since the temperature of the surroundings is greater than that of the sphere, the energy per second, $Q$, gained from the surroundings is given by

$$Q = \sigma 4\pi r^2 (300^4 - 200^4).$$

The mass $m$ of the sphere = volume x density = $\frac{4}{3}\pi r^3 \rho$, where $\rho$ is the density.
If $C$ is the specific heat capacity of the metal, and $\theta$ is the initial rise per second of its temperature, then

$$Q = mc\theta = \frac{4}{3} \pi r^3 \rho C \theta = \sigma 4 \pi r^2 (300^4 - 200^4)$$

Simplifying,

$$\theta = \frac{\sigma (300^4 - 200^4) x 3}{r \rho C}$$

$$= \frac{5.7 \times 10^{-8} x (300^4 - 200^4) x 3}{30 \times 10^{-3} x 8000 x 400}$$

$$= 0.012 \text{ K}^{-1}$$

**Limitations of Wien’s and Rayleigh-Jeans Radiation Formulae**

**Limitations of Wien’s radiation Formula**

The harmony between theory and experiment did not last long in the Wien’s formular. To Planck’s consternation, experiments performed in Berlin showed that the Wien’s law did not correctly describe the spectrum at very low frequencies but explains the experimental results fairly well for low values of $\lambda T$.

That is, Wien derived the law of energy distribution in the blackbody spectrum proceeding from clasical concepts. However, as was soon made clear, the formula of Wien’s radiation law was correct only in the case of short ( in relation to the intensity maximum ) waves, his expression was invalid at high temperatures and long wavelength. Nevertheless, the laws of Wien have played a considerable part in the development of quantum theory.

**Limitations of Rayleigh-Jeans Law**

Lord Rayleigh in 1900 applied the principle of equipartition of energy to the electromagnetic vibrations. J.H Jeans also contributed to the experiment, by his attempt to the deduction of a formula for energy per unit volume inside an enclosure with perfectly reflecting walls.
The law showed that the energy density, $U\nu d\nu$ i.e. the amount of energy per unit volume of the enclosure in the frequency range from $\nu$ to $\nu + d\nu$ is given by
$$U\nu d\nu = \frac{8\pi\nu^2 kT}{C^3} d\nu,$$
where $k$-Boltzmann’s constant and $c$ – Speed of light in free space.

The Rayleigh-Jeans formula can be transformed in terms of the wavelength $\lambda$ by using the relation, $\nu = \frac{C}{\lambda}$ and $d\nu = -\frac{C}{\lambda^2} d\lambda$.

The energy $U\nu d\nu$ contained in a frequency interval between $\nu$ and $\nu + d\nu$ is equal to that contained in a corresponding wavelength interval between $\nu$ and $\nu + d\nu$ and an increase in frequency corresponds to a decrease in $\lambda$.

Therefore: $U\lambda d\lambda = -U\nu d\nu$

$$= -\frac{8\pi (C/\lambda)^2}{C^3} K T \left(-\frac{C}{\lambda^2}\right) d\lambda = -\frac{8\pi kT}{\lambda^4} d\lambda$$

This equation is another form of the Rayleigh-Jeans law

Recall the following:

- **Wien’s displacement formula/Law:** $\lambda_m T = \frac{h}{2.898 \times 10^{-3} mK}$,

which is obtained by finding $\frac{dU}{d\lambda} \bigg|_{\lambda_m = 0}$ where $U_\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda kT} - 1}$

It is used to determine the temperature of a black body by determining the wavelength $\lambda_m$ at which the intensity of the radiation is maximum.

- **Stefan – Boltzman law:** $E = \sigma T^4$ where $\sigma = \frac{2\pi^5 k^4}{15h^3 c^2} = 5.67 \times 10^{-8} \text{ Wm}^{-2} \text{K}^{-4}$,

derived from $E = \frac{cU}{4}$ where $U = -\frac{4}{\sigma} T^4$ and $U = \int_0^\infty U_\lambda d\lambda = \int_0^\infty \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{e^{hc/\lambda kT} - 1} d\lambda$
The work of Wien, Rayleigh and Jeans left the physics world in a dilemma until Max Planck made a radical change in classical physics by introducing the concept of quantization.

Rayleigh-Jeans Law explains the experimental facts for very long wavelengths but not for shorter wavelengths. According to the law, as $\lambda$ decreases the energy density $U_\lambda$ will continually increase, and as $\lambda$ tends to zero, $U_\lambda$ approaches infinity. This is contrary to experimental results. The law leads to an absurd result $U = \infty$, which shows that for a given quantity of radiant energy, all the energy will finally be confined in vibrations of very small wavelengths.

$$U = \int_0^\infty \frac{8\pi KT}{\lambda^4} d\lambda = 8\pi KT \left[ -\frac{1}{3\lambda^3} \right]_0^\infty = \infty$$

But experimental results show that $U_\lambda d\lambda \to 0$ as $\lambda \to 0$. This discrepancy between theoretical conclusion and experimental result is sometimes known as “ultraviolet catastrophe”. It is mainly because of the assumption that energy can be absorbed or emitted by the atomic oscillators continuously in any amount.

**Planck’s Theory of radiation**

The failure of Wien's and Rayleigh-Jeans classical approach to provide satisfactory explanation of the distribution of energy in the spectrum of a black body led Max Planck to propose the quantum hypothesis/theory of radiation.

He assumed that the atoms in the walls of a black body behave like simple harmonic oscillators, and each has a characteristics frequency of oscillation. The following assumptions about the atomic oscillator were also made:

1. A simple harmonic oscillator cannot have any arbitrary values of energy but only those values of the total energy $E$ that are given by the relation $E = nh\nu$, where $n = 0,1,2,3, \ldots \ldots \ldots , n$ is called the quantum number, $\nu$ is the frequency of oscillation, and $h = 6.626 \times 10^{-34}$ Js is Planck's constant. In this relation, $h\nu$ is the basic unit of energy and is called a quantum of energy. Thus the relation shows that the total energy of an oscillator is quantized.
2. As long as the oscillator has energy equal to one of the allowed values given by the relation \( E = n \hbar \nu \), it cannot emit or absorb energy. Therefore, the oscillator is said to be in a stationary state or a quantum state of energy.

The emission or absorption of energy occurs only when the oscillator jumps from one energy state to another.

If the oscillation jumps down from a higher energy state of quantum number \( n_2 \) to a lower energy state of quantum number \( n_1 \), the energy emitted is given by

\[ E_2 - E_1 = (n_2 - n_1) \hbar \nu \]

If \( n_2 - n_1 = \text{unity} \), then \( E_2 - E_1 = \hbar \nu \)

Similarly, an oscillator absorbs a quantum \( \hbar \nu \) of energy when it jumps up to its next higher energy state.

According to Planck, the quantum theory is applicable only to the process of emission and absorption of radiant energy. In 1905, Einstein extended Planck’s quantum theory by assuming that a monochromatic radiation of frequency \( \nu \) is consists of a stream of photons each of energy \( \hbar \nu \) and the photons travel through space with the speed of light.

**Planck’s Radiation Law**

On the basis of the quantum theory, Planck obtained the formula for the average energy of an oscillator.

\[ E = \frac{\hbar \nu}{e^{\frac{\hbar \nu}{kT}} - 1} \]

Then assuming that the average value of the energies of the various modes of oscillation in black body radiation is given by the above, Planck obtained the equation.

\[ U_\nu d_\nu = \frac{8\pi \hbar \nu^3}{C^3} \cdot \frac{1}{e^{\frac{\hbar \nu}{kT}} - 1} d\nu \]

Where \( U_\nu \) \( d_\nu \) is the energy per unit volume in the frequency range \( \nu \) and \( \nu + d\nu \). In terms of the wavelength of the radiation, the last equation becomes
\[ U_\lambda d_\lambda = \frac{8\pi hc}{\lambda^5} \cdot \frac{1}{e^{h\nu/kT} - 1} d\lambda , \{ \text{Since } \nu = \frac{c}{\lambda} \text{ and } d\nu = \frac{-c}{\lambda^2} d\lambda \} \]

The last two equations are the two forms of Planck’s radiation law. From Planck’s law in the form \( U_\lambda d_\lambda \), the Rayleigh- Jeans law, Wiens law and the Stefan – Boltzmann formula are obtained as mathematical consequences. The success of Planck’s hypothesis was the beginning of quantum mechanics.